

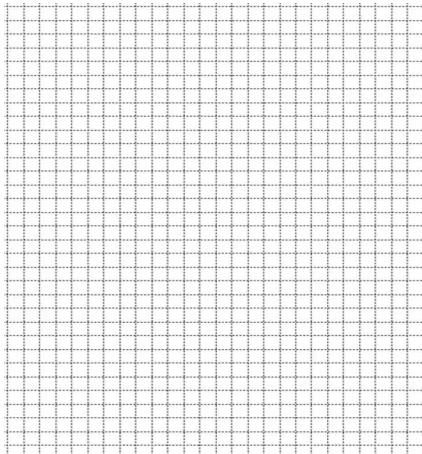
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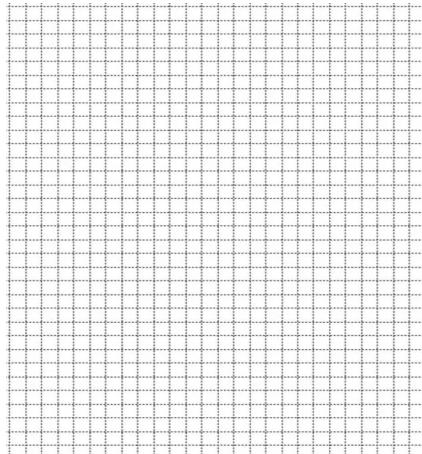
**Math 12 Honours: Section 5.4 Graphing Exponential & Logarithmic Equations with Transformations**

1. For each graph below, find the Y-intercept, X-intercept (if any), Domain and Range, and Asymptotes. Then graph the function with the grid provided. Be sure to label the axis on your grid.

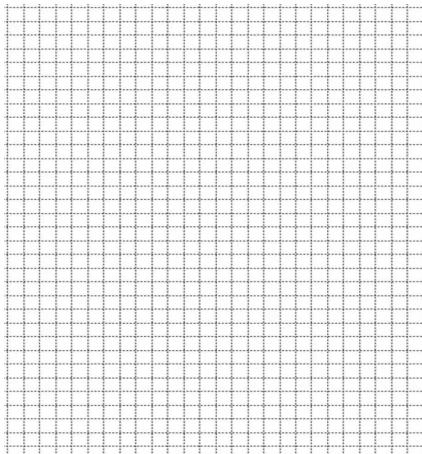
a)  $y = 3(2)^{x-2} - 3$



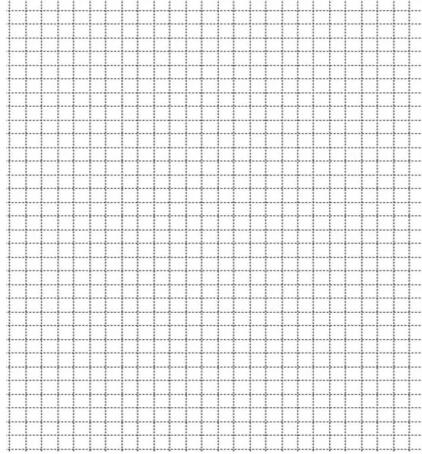
b)  $y = -0.5(3)^{-x} + 1$



c)  $y = -2(\overline{0.33})^{2-x} + 4$

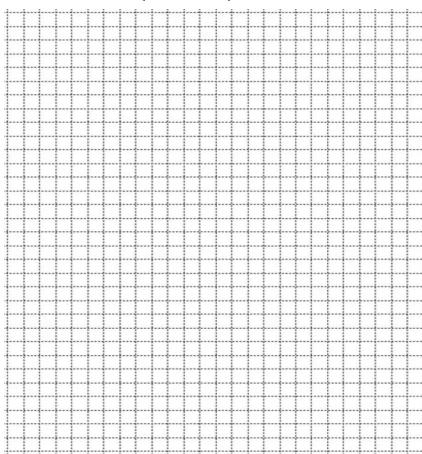


d)  $y = 4(81)^{0.25x-1}$

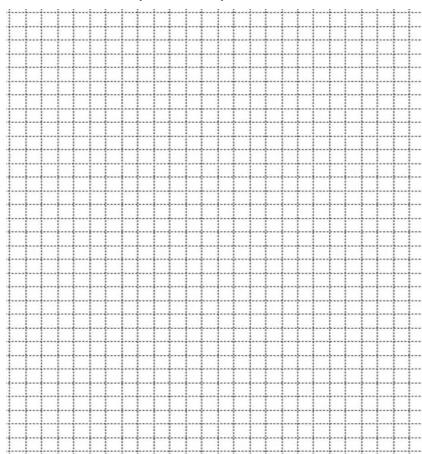


2. Graph each of the following logarithmic functions. Indicate the Domain, Range, equations of Asymptotes, and any intercepts. Be sure to label your axis on the graph:

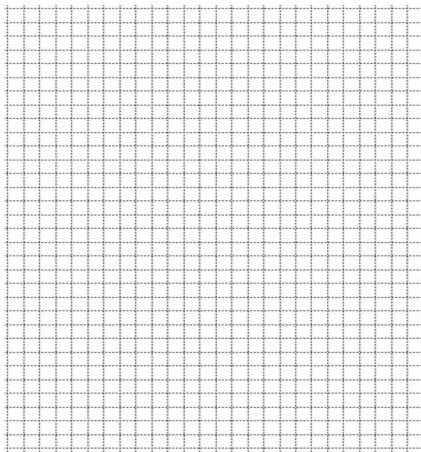
a)  $y = \log(2x-3) + 1$



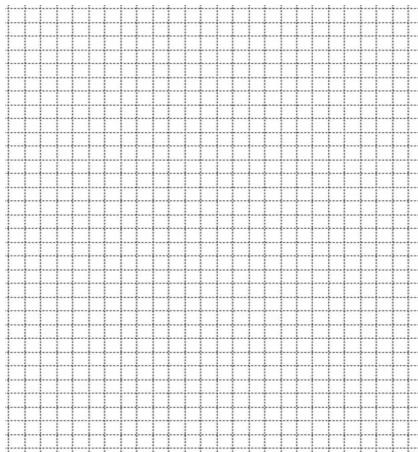
b)  $y = \log(4x+1) - 3$



c)  $y = \log_2(2 - x) + 4$

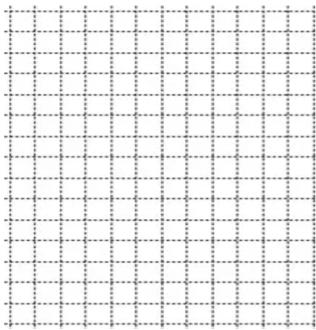


d)  $\log y = -\log_3(3x) - 5$

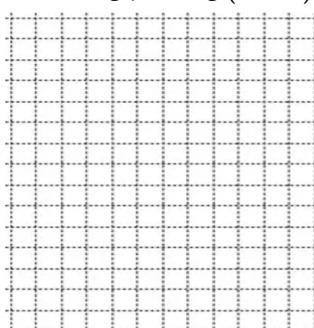


3. Graph the following functions. Indicate the domain and range:

a)  $\log y = 2 \log x$



b)  $0.5 \log y = \log(x - 2)$



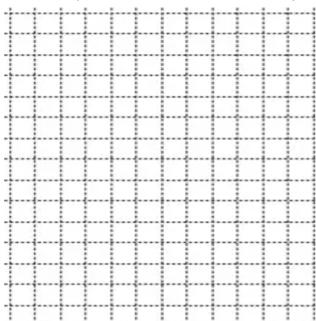
c)  $\log y = \log(\sin x)$



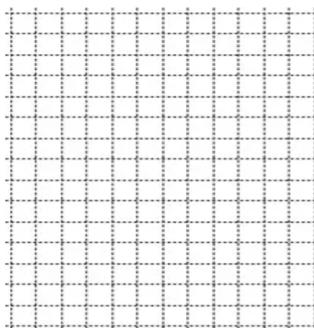
d)  $\log y = \log(\sec x)$



$\log_y(x^3 + 3x^2 + 3x + 1) = 3$



$y = \log(\log x)$



4. Given that  $f(x) = \log_3(x + 2) - 4$ , find  $f^{-1}(x)$ , the inverse function of  $f(x)$

5. What transformation is required to go from  $y = \log x$  to  $y = \log\left(\frac{1}{x}\right)$ ?

6. What transformation is required to go from  $y = \log_3 x$  to  $y = \log_3 \frac{4}{x}$ ?

7. Are the following graphs the same? Yes or NO? Explain:

a)  $y = 4(0.5)^x$  and  $y = 4(2)^{-x}$       b)  $y = 24(0.5)^{2-x}$  and  $y = 6(2)^x$

8. What is the inverse function of  $f(x) = 3(5)^{x-2}$ ? What are the domain, range, x-intercepts, and Y-intercepts of both  $f(x)$  and  $f^{-1}(x)$ ? What patterns do you notice?

9. Find the coordinates of the points of intersection of the graphs:  $y = \log_{10}(x-2)$  and  $y = 1 - \log_{10}(x+1)$  (Euclid)

10. Determine all the points where the two functions intersect:  $y = \log_{10} x^4$  and  $y = (\log_{10} x)^3$  (Euclid)

11. If  $\log(a+b) = x$  and  $\log(a^2 - ab + b^2) = y$ , then what is the value of  $\sqrt[4]{a^3 + b^3}$  in terms of "x" and "y"?

12. Solve the inequality

a)  $\log_4(2x-3) > 2$

b)  $\log_{\frac{1}{4}} x > 5$

13. What is the domain of the following functions:

i)  $y = \log_{0.5}(\log_5 x)$

ii)  $y = \log_{0.5}(\log_5(\log_{\frac{1}{3}} x))$

14. Solve for "x":  $\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$  (Euclid)

15. Find all the values of "x" such that:  $\log_{2x}(48\sqrt[3]{3}) = \log_{3x}(162\sqrt[3]{2})$  (Euclid)

16. Determine all real numbers "x" for which:  $2\log_2(x-1) = 1 - \log_2(x+2)$  (Euclid)

17. Determine all pairs of angles  $(x,y)$  with  $0^\circ \leq x < 180^\circ$  and  $0^\circ \leq y < 180^\circ$  that satisfy the following systems of equations: (Euclid)

$$\log_2(\sin x \cos y) = -\frac{3}{2} \quad \text{and} \quad \log_2\left(\frac{\sin x}{\cos y}\right) = \frac{1}{2}$$

18. Determine all pairs  $(a,b)$  of real numbers that satisfy the following systems of equations: Give your answer in simplified exactd form: (Euclid)

$$\sqrt{a} + \sqrt{b} = 8 \quad \text{and} \quad \log_{10} a + \log_{10} b = 2$$

19. Solve for "x"  $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$  (AIME)

## Problem

The value of  $x$  that satisfies  $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$  can be written as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

## Solution

Let  $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020} = n$ . Based on the equation, we get  $(2^x)^n = 3^{20}$  and  $(2^{x+3})^n = 3^{2020}$ . Expanding the second equation, we get  $8^n \cdot 2^{xn} = 3^{2020}$ . Substituting the first equation in, we get  $8^n \cdot 3^{20} = 3^{2020}$ , so  $8^n = 3^{2000}$ . Taking the 100th root, we get  $8^{\frac{n}{100}} = 3^{20}$ . Therefore,  $(2^{\frac{3}{100}})^n = 3^{20}$ , and using the our first equation( $2^{xn} = 3^{20}$ ), we get  $x = \frac{3}{100}$  and the answer is 103. ~rayfish