Name:______________________________ Date:___________________________

Math 12 Honours: Section 5.4 Graphing Exponential & Logarithmic Equations with Transformations

1. For each graph below, find the Y-intercept, X-intercept (if any), Domain and Range, and Asymptotes. Then graph the function with the grid provided. Be sure to label the axis on your grid.

a) $y = 3(2)^{x-2} - 3$

b) $y = -0.5(3)^{-x} + 1$

c) $y = -2\left(0.\overline{33}\right)^{2-x} + 4$

d) $y = 4(81)^{0.25x-1}$

2. Graph each of the following logarithmic functions. Indicate the Domain, Range, equations of Asymptotes, and any intercepts. Be sure to label your axis on the graph:

a) $y = \log(2x-3)+1$

b) $y = \log(4x+1)-3$

c) $y = \log_2(2-x)+4$	d) $\log = -\log_3(3x)-5$

3. Graph the following functions. Indicate the domain and range:

a) $\log y = 2\log x$	b) $0.5\log y = \log(x-2)$
c) $\log y = \log(\sin x)$	d) $\log y = \log(\sec x)$
$\log_y(x^3+3x^2+3x+1)=3$	$y = \log(\log x)$

4. Given that $f(x) = \log_3(x+2)-4$, find $f^{-1}(x)$, the inverse function of $f(x)$

5. What transformation is required to go from $y = \log x$ to $y = \log\left(\frac{1}{x}\right)$?

6. What transformation is required to go from $y = \log_3 x$ to $y = \log_3 \frac{4}{x}$?

7. Are the following graphs the same? Yes or NO? Explain:

a) $y = 4(0.5)^x$ and $y = 4(2)^{-x}$

b) $y = 24(0.5)^{2-x}$ and $y = 6(2)^x$

8. What is the inverse function of $f(x) = 3(5)^{x-2}$? What are the domain, range, x-intercepts, and Y-intercepts of both $f(x)$ and $f^{-1}(x)$? What patterns do you notice?

9. Find the coordinates of the points of intersection of the graphs: $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$ (Euclid)

10. Determine all the points where the two functions intersect: $y = \log_{10} x^4$ and $y = (\log_{10} x)^3$ (Euclid)

11. If $\log(a+b) = x$ and $\log(a^2 - ab + b^2) = y$, then what is the value of $\sqrt[4]{a^3 + b^3}$ in terms of “x” and “y”?

12. Solve the inequality

a) $\log_4(2x-3) > 2$

b) $\log_{\frac{1}{4}} x > 5$

13. What is the domain of the following functions:

i) $y = \log_{0.5}(\log_5 x)$

ii) $y = \log_{0.5}\left(\log_5\left(\log_{\frac{1}{3}} x\right)\right)$

14. Solve for "x": $\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$ (Euclid)

15. Find all the values of "x" such that: $\log_{2x}\left(48\sqrt[3]{3}\right) = \log_{3x}\left(162\sqrt[3]{2}\right)$ (Euclid)

16. Determine all real numbers "x" for which: $2\log_2(x-1) = 1 - \log_2(x+2)$ (Euclid)

17. Determine all pairs of angles (x,y) with $0° \le x < 180°$ and $0° \le y < 180°$ that satisfy the following systems of equations: (Euclid)

$\log_2(\sin x \cos y) = -\frac{3}{2}$ and $\log_2\left(\frac{\sin x}{\cos y}\right) = \frac{1}{2}$

18. Determine all pairs (a,b) of real numbers that satisfy the following systems of equations: Give your answer in simplified exactd form: (Euclid)

$\sqrt{a} + \sqrt{b} = 8$ and $\log_{10} a + \log_{10} b = 2$

19. Solve for "x" $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ (AIME)

Problem

The value of x that satisfies $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Solution

Let $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020} = n$. Based on the equation, we get $(2^x)^n = 3^{20}$ and $(2^{x+3})^n = 3^{2020}$. Expanding the second equation, we get $8^n \cdot 2^{xn} = 3^{2020}$. Substituting the first equation in, we get $8^n \cdot 3^{20} = 3^{2020}$, so $8^n = 3^{2000}$. Taking the 100th root, we get $8^{\frac{n}{100}} = 3^{20}$. Therefore, $(2^{\frac{3}{100}})^n = 3^{20}$, and using the our first equation($2^{xn} = 3^{20}$), we get $x = \frac{3}{100}$ and the answer is $\boxed{103}$. ~rayfish